

Futures Markets -- Chapter 2: Futures Contract Pricing

5. Suppose a portfolio manager observes the following prices and rates on April 11th

Bid price on IBM stock	\$139.00	
Ask price on IBM stock	\$140.10	
September 15 th S&P 1 futures bid price	\$143.60	
September 15 th S&P 1 futures ask price	\$143.70	
Bid price on a 157-day, \$1 million face value T-Bill	\$959,816.00	
Ask price on a 157-day, \$1 million face value T-Bill	\$960,218.00	
Arbitrageur overnight borrowing rate		10.50%
Arbitrageur overnight lending rate		9.00%
Transactions fees (per-share basis)		\$0.020
Stocks		\$0.012
Futures		\$0.002
Borrowing or lending		\$0.006
Dividend on IBM (paid first day of January, April, July and October)		\$2.50

a. What are the implied repo and reverse repo rates?

The first task is to determine the futures values of the dividends. A dividend will be paid on July 1, which is 76 days before September 15th. **For an implied repo rate the lending rate is the relevant rate of:**

$$FV(\text{Dividends}) = \$2.50(1 + (.09)(76/360)) = \$2.55 \text{ [carrying cost]}$$

For an implied reverse repo rate the borrowing rate is the relevant rate of:

$$FV(\text{Dividends}) = \$2.50(1 + (.105)(76/360)) = \$2.56 \text{ [carrying cost]}$$

The repo rate then is determined by:

Buy IBM stock, sell short the S&P 1 futures contract, include dividend as income, subtract out transactions fee .

$$= (\$143.60 + \$2.55 - \$0.02 - \$140.10) = 4.30\% \text{ for 157 days}$$

$$\implies 360/157 (4.30\%) = 9.859 \text{ or } 9.86\%$$

The reverse repo rate will be calculated as:

Sell short IBM stock, buy to go long S&P 1 futures contract, include dividend and subtract transactions fee.

IBM is shorted at the bid price of \$139.90, the value of the futures is the ask price including transaction fees and the future dividend

$$= (\$143.70 + \$0.02 + \$2.56 - \$139.90)/139.90 = 4.56\% \text{ for 157 days}$$

$$\implies (360/157) (4.56\%) = 10.46\%$$

b. What are the no-arbitrage upper and lower bounds for pure arbitrage?

The arbitrageur's annualized borrowing rate is 10.50%, so the relevant borrowing rate in this instance is:

$$(157/360) (10.50\%) = 4.58\% \text{ for 157 days}$$

The lending rate is the greater of the overnight rate of $(157/360) (9\%) = 3.93\%$ for 157 days, or the T-Bill lending rate. Since the arbitrageur would be buying T-Bills, the appropriate price to use is the ask price. Consequently, the T-Bill rate would be:

$$\text{T-Bill Rate} = (\$1,000,000 - \$960,218)/\$960,218 = 4.14\% \text{ for 157 days}$$

$$\implies (360/157) (4.14\%) = 9.49\%$$

The lower bound would turn out to be:

$$= \$139.90(1.0414) - \$0.02 - \$2.56 = \$143.11$$

And the upper bound would be:

$$= \$140.10(1.0458) + \$0.02 - \$2.55 = \$143.99$$

assuming full use of the short sale proceeds.

c. Is there any arbitrage opportunity? Examine both the no-arbitrage upper and lower bounds, and the synthetic borrowing and lending rates.

For the cash and carry arbitrage the implied repo rate would be:

$$\text{Repo rate} - \text{Borrowing Rate} = 9.86\% - 10.5\% \Rightarrow \text{no arbitrage possibility}$$

For the reverse cash and carry arbitrage the implied reverse repo rate would be:

$$\text{Lending Rate} - \text{Reverse Repo Rate} = 9.49\% - 10.46\% \Rightarrow \text{no arbitrage possibility}$$

Alternatively, both the futures bid price of \$143.60 and the futures ask price of \$143.70 for the S&P 1 contract lie between the lower and upper bounds for no arbitrage, i.e.,

$$\$143.11 \text{ -----} \$143.99$$

6. Suppose a corporate treasurer faces the same prices and rates as those in Problem 5. On April 11th, the treasurer is considering purchasing \$100M face value T-Bills expiring on September 15th.

a. What rate of return will the treasurer earn if he buys the T-Bills?

The rate will be the same as in 5 b above:

$$\text{T-Bill Rate} = (\$1,000,000 - \$960,218) / \$960,218 = 4.14\% \text{ for 157 days}$$

$$\Rightarrow (360/157) (4.14\%) = 9.49\%$$

b. and c. What rate of return will he earn if he uses a cash-and-carry synthetic lending strategy using IBM stock? Is this a cash and carry or reverse cash and carry arbitrage situation? Will this strategy be profitable? Calculate Q.

A corporation that is planning to lend might be able to earn a higher return by doing so through a cash-and-carry strategy through the spot and futures markets. These types of operations are called quasi-

arbitrage strategies. Since T-Bills are discounted securities, this will be a cash-and-carry synthetic arbitrage opportunity if it exists.

When determining the carrying costs for the cash and carry synthetic lending transaction, we will need to look at the transactions fees, the relevant transactions fees would be:

$TF = \$0.012 + \$0.002 - \$0.006 = \0.008 which we can use in evaluating our cash-and-carry repo rate:

Buy IBM, Short S&P 1 futures, pay transactions fees, record dividends earned and invested

Repo Rate = $(\$143.60 - \$0.008 + \$2.55 - \$140.10)/\$140.10 = 4.31\%$ for 157 days

$\Rightarrow (360/157) (4.31\%) = 9.88\%$

From this result we can ascertain that there is a quasi –arbitrage opportunity because:

the T-Bill rate of return on an annualized basis is: $9.49\% < 9.88\%$ the return on synthetic lending.

Alternatively, we can potential arbitrage profit, $Q = \$143.60 - (\$140.10(1.0414) + \$0.008 - \$2.55)$.

$Q = \$0.24 > 0 \Rightarrow$ an arbitrage opportunity exists.

7. Suppose a portfolio manager faces the same prices and rates as those in Problem 5. On April 11, she is considering selling stock and buying T-bills with \$100M Face Value that expire on September 15th.

a. What alternative strategy could the portfolio manager pursue to achieve a similar goal using futures?

In this case, the appropriate strategy would be to hold the IBM stock and sell the S&P 1 futures contracts. This will lock in the price of the IBM stock on September 15th -- \$143.70

b. Construct a quasi-arbitrage table to analyze the choice between selling stocks and buying T-bills using a synthetic strategy. Compute Q and determine which strategy is better.

The quasi arbitrage table is as follows:

Benchmark position: Sale of Asset

Cash-and-Carry Pure Arbitrage	Cash-and-Carry Quasi-Arbitrage	Relevant Price or Rate
Components	Components	
1. Buy Spot Receive Payouts	Refrain from Selling	Bid
	Receive Value/Pay Storage	
2. Short Futures	Short Futures	Bid
3. Borrow	Borrow	Borrowing Rate

Resulting position: synthetic sale of asset

Transactions fees: $TF = -TF_1 + TF_2 - TF_3$

1. Save TF_1
2. Pay TF_2
3. Save TF_3

Consequently, $TF = -.012 + .002 - .006$

Because she saves the transactions fees associated with selling the stock and buying the T-Bills, but pays the costs associated with going short the futures.

Therefore,

$$\Omega = F_{t,T} - \{ P_t (1 + r_{t,T}(\text{T-bill ask})) + TF - FV(\text{Dividend}) \}$$

$$= 1.43.60 - (\$139.90(1.0414) - .016 - 2.55) = \$0.47 > 0 \implies \text{an arbitrage opportunity exists}$$

c. How does the value of Ω compare with that in Problem 6? How do you explain this difference?

This value of Ω exceeds the value of $\Omega = \$0.24$ from problem 6. This occurs for two reasons, one the relevant spot price for the corporate treasurer was the foregone bid [\$139.90], while the relevant spot price for the manager is ask [\$140.10]. Secondly, the effective transactions fees are lower for the treasurer than in the case of the manager.

10. Consider platinum and palladium. The T-bill yields for this period are approximately 7.4%. Storage costs for these metals are minimal. Are these metals pure assets or convenience assets? (Note that a metal can be a pure asset during part of the year and a convenience asset the rest of the year.)

We can compute the % rate of price change between successive contracts [using settlement prices]:

Platinum:

October 1988 to January 1989 =====> $[519.40 - 520.40]/520.40 = -.19\%$ for 3 months

Or $4 \times -.19\% = -.76\%$ a year

January 1989 to April 1989 =====> $[522.90 - 519.40]/519.40 = +.67\%$ for 3 months

Or $4 \times .67\% = 2.68\%$ a year

April 1989 to July 1989 =====> $.90\%$ for 3 months or $4 \times .90\% = 3.60\%$ a year

July 1989 to October 1989 =====> 1.08% for 3 months or $4 \times 1.08\% = 4.32\%$ for a year

Palladium:

December 1988 to March 1989 =====> $[121.65 - 122.50]/122.50 = -.69\%$ for 3 months

Or $4 \times -.69\% = -2.76\%$ a year

March 1989 to June 1989 =====> $-.82\%$ for 3 months or $4 \times -.82\% = -3.28\%$ for a year.

A major difference between assets that are readily lent out for short sales and those that are not lies in the reasons the investor holds the assets. In a short sale, the lender gives up the commodity or security [e.g. stock] now and receives it back intact in the future. The borrower is required to compensate the lender for any explicit payouts, such as dividends or interest, associated with holding the asset. As long as the lender does not need the asset of any reason during the duration of the short, lending for a short sale is equivalent to holding the asset over the same period.

When an investor can sell an asset short between two dates t and T , thereby agreeing to pay the explicit payouts, and compensate the buyer where the commodity or security is held strictly for investment purposes, the assets are called **pure assets**. [financial securities are examples of pure assets].

Other assets that are held for the physical services they provide, as well as, for potential investment return are called **convenience assets**. Commodities are frequently referred to as convenience assets. For example a corn processor, such as ADM might hold corn to avoid having to shut down a plant in the event of a sudden corn shortage.

None of these contracts seem to be priced as if the markets are at full carry. Even the later months for platinum have a rate of price increase that is less than the T-Bill rate, and so consequently, these metals appear to be convenience assets.